
ABSTRACT

With the recent breakthrough in the field of optical frequency measurement and synthesis, it is now possible to accurately measure the frequency of optical frequency signals, generate arbitrary waveform light pulses and perform accurate optical frequency synthesis. Generating precise optical frequencies with a functional power is necessary in many fields of science and technology. This paper reviews different methods for generating the optical frequency comb generator (OFCG). This article highlights the different mechanisms integral to the operation of the pulsed laser and the tailored optical fiber. It also presents the techniques that enable the promised frequency synthesis and measurement. Three different techniques discussed in this paper which is 1) EO Phase Modulator, 2) Mode locked Laser and 3) Fiber Ring based Comb generator.

KEYWORDS: Optical frequency comb generator, Electro-Optic phase modulation, Mode locked laser, Fiber ring based optical frequency comb generator.

INTRODUCTION

Researchers working with the optical part of the electromagnetic spectrum (10¹⁴-10¹⁵ Hz) have wished to have coherent instrumentation that can perform similar operations to the instruments available for the radio-frequency and microwave parts of the spectrum (0 Hz -10¹¹ Hz). The difference between two laser frequencies can be measured easily by superimposing the two laser beams on a photo detector and monitoring a beat signal. It is possible to directly count the frequency differences up to the order of 100GHz using commercial fast photodiodes and microwave frequency counters [7].

As the frequency gap between the endpoint of a harmonic laser frequency chain and an unknown optical frequency to be measured can easily range up to the hundreds of THz, there has been a greater interest in measuring the differences of much larger optical frequencies.

Optical frequency comb generator (OFCG) is yet another compact setup to measure frequency gaps on the order of a few THz based on very efficient creation of side bands in a large index electro optic phase modulator. Further side bands are created and the side bands are in resonance if the modulation frequency matches the free spectral range of the optical resonator



Figure 1: A typical situation in frequency metrology, a well-known reference frequency and an unknown frequency tens or hundreds of THz apart.

Optical frequency combs is a unique technique for ultra-precise measurements of time and frequency. However the challenge is mainly due to requirements of very larger comb frequency spacing, nonlinear frequency conversion and broadening with sufficient power. Optical Frequency Comb Generator (OFCG) having multiple applications such as Wavelength Division Multiplexing (WDM), spectroscopy and Optical arbitrary waveform generation. We get a broad and amplitude flattened comb with an equal spacing in Optical arbitrary waveform generation. J.L. Hall at National Institute of Standard and Technology and T.W. Hansch at Max-Plank Institute were awarded with the Nobel Prize in Physics “for their contribution to the development of laser-based spectroscopy, including the Optical frequency comb technique”.

RELATED WORK

Photonics is widely seen as a rapidly expanding interdisciplinary field having numerous applications from consumer products to defence and space hardware development. Fiber optic communication is one such application which enabled development of a variety of optical components and technologies.

The Optical frequency comb generator (OFCG) produces a spectrum of evenly spaced discrete comb lines, centred at optical carrier frequency. These comb lines are spaced according to

$$f(n) = f_0 + n \cdot f_r$$

where n is an integer, f_r is the comb spacing and f_0 is the carrier frequency. The optical frequency comb generator (OFCG) has promising applications in optical metrology, frequency chain generation, optical atomic clocks, wavelength/ frequency division multiplexing in optical communication [2] and high precision spectroscopy [3]. Frequency comb source can be generated by using different mechanisms such as modulation of a continuous wave laser, stabilization of the pulse train generated by mode locked laser and using ring resonator approach etc. The mode-locked laser based OFCG has fixed comb spacing due to its cavity length with stability depending upon mode locking condition. The OFCG with adjustable central frequency and comb spacing can be made by modulating continuous wave laser with a RF source scheme generates few comb lines having unequal amplitude, which can be made flat by cascading few modulation stages or using a feedback loop with optical amplification.

DIFFERENT TYPES OF ALGORITHM

EO Phase Modulator based comb Generator

Optical Frequency comb generator can be generated by using phase modulation and Electro Optic (EO) phase modulation. Basic principle of an Electro-Optic modulator is change in refractive index or change in birefringence based on electrically induced field. It depends on the device configuration of an Electro-Optic modulator, the following properties of light wave varied in controlled ways: phase, polarization, amplitude, frequency, or direction of propagation [2]. A light wave can be modulated with respect to the change in the index of refraction of electro-optic material. The applied voltage around the electro-optic material causes the change in refractive index. The depth of modulation depends on the modulating signal parameters amplitude (voltage) and frequency. The schematic explains the generation of optical frequency comb by using electro-optic phase modulator.

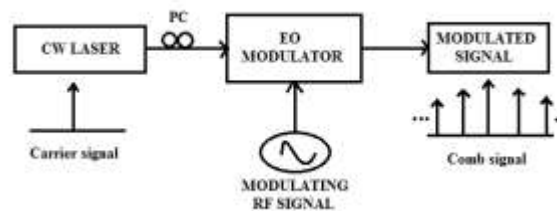


Figure 2: Schematic of Electro-Optic phase modulation

Let us assume input optical signal electrical field is

$$E_{ix} = E_i \cos(\omega t \pm \phi_0) \quad (1)$$

The output of the modulated optical signal at the end of the modulator represented as

$$E_o(t) = E_i \cos(\omega t \pm \phi) \quad (2)$$

Where,

$$\phi = \phi_0 + \Delta\phi_x \Rightarrow \frac{2\pi}{\lambda} (n_0 + \Delta n_x) L \quad (3)$$

Φ represents the total modulated phase shift, Φ_0 is initial phase and $\Delta\Phi_x$ is the change in phase due to modulation.

The induced phase shift $\Delta\Phi_x$ can be written as in terms of electric field of modulating signal

$$\Delta\phi_x = \left(\frac{\pi}{\lambda}\right) n_x^3 r L E \quad (4)$$

Where L is the length of the electro-optic material of width d and E is the electric field generated around the electro-optic material due to applied modulating signal of voltage V . The electric field of modulating signal can be written as

$$E = E_m \sin(\omega_m t) \quad \& \quad E = \frac{V}{d} \quad (5)$$

$$\Delta\phi_x = \pi \left(\frac{V}{V_\pi} \right) \quad (6)$$

Induced phase

Where V is applied voltage and V_π is half wave voltage of a given modulator configuration. Total modulated phase is

$$\phi = \left(\frac{2\pi}{\lambda} \right) n_x L \pm \delta \sin(\omega_m t) \quad (7)$$

δ is called depth of modulation index. By neglecting the constant phase term and applying the identity, the output of modulated light wave becomes

$$\begin{aligned} E_o(t) = E_i & (J_0(\delta) \cos(\omega t) + J_1(\delta) \cos(\omega + \omega_m)t - \\ & J_1(\delta) \cos(\omega - \omega_m)t + J_2(\delta) \cos(\omega + 2\omega_m)t - \\ & J_2(\delta) \cos(\omega - 2\omega_m) + \dots) \end{aligned} \quad (8)$$

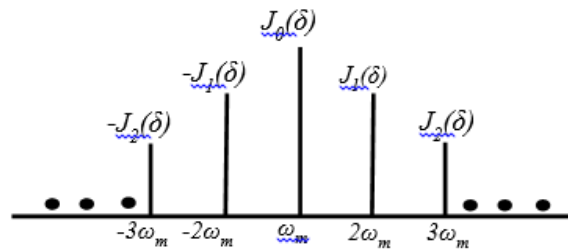


Figure 3: Spectrum of the composed carrier with an amplitude $J_0(\delta)$ and a set of sidebands spaced symmetrically on either side of the carrier at frequency separation $\omega_m, 2\omega_m, 3\omega_m, 4\omega_m \dots$ etc

The spectrum is composed of a carrier with an amplitude $J_0(\delta)$ and a set of sidebands spaced symmetrically on both side of the carrier at frequency separation $\omega_m, 2\omega_m, 3\omega_m, 4\omega_m \dots$ etc. The function $J_n(\delta)$ is called Bessel function of first kind and order n . The amplitude of the spectral components depend on Bessel function $J_n(\delta)$.

The difference between the two closest frequencies at which modulation index (δ) falls to 50 percentage of the maximum value or the spectrum within 3dB is called Bandwidth of the comb spectrum. The numbers of spectral lines are linearly related with RF voltage deriving the phase modulator ($\delta \propto V_m$). RF power limits the number of lines generated by a single phase modulator. The bandwidth of combs however is limited. Commercially available phase modulator having low $V_\pi \sim 3V$ and RF signal power limits 1W (RF voltage $\sim 7V$). It limits the number lines 20 within 3 dB bandwidth. Carson's rule also explaining the bandwidth of the modulation is equal to $2(\delta+1)$ and modulation index is proportional to RF voltage. The bandwidth of a comb spectrum was limited due to the limitation of low V_π and power of the RF signal. To generate large bandwidth, we would have to cascade phase modulators in series, but this process was expensive and inefficient. By using feedback mechanism we can generate broadband comb signal, it was cost effective (inexpensive) and efficient also [1].

Femtosecond Frequency Comb Generation

In 1991 the discovery of Kerr-lens mode-locking in Wilson Sibbett's laboratory at the University of St. Andrews [7] led to a reliable and simple method for producing pulses of laser radiation in the near infrared with a duration of just a few tens of femtoseconds: in other words each light pulses has just a few cycle of light wave. This remarkable advance has been the enabling technology for a whole raft of new types of measurements in an exceptionally wide range of fields. In the first flash lamp pumped Nd:glass and Nd:YAG mode-locked laser appeared in the mid-sixties with less than 100 pico second in duration and demonstrated one of the most powerful interference phenomena of the nature. CW operation of dye lasers with broad bandwidth triggered the second generation of mode-locked lasers. Optical pulses shorter than 1 pico second could be produced and improvements in the cavity design allowed breaking of the 100-femto second barrier. In 1984 Intracavity dispersion control by means of Brewster angled prism pairs was the next major breakthrough [8]. The development of new solid state laser materials led to the emergence of third generation laser sources with the discovery of self-mode locking in Ti:sapphire laser, the explanation as per the Kerr-lens mode-locking and development of the design to produce 10-femto second pulses. Recently pulses shorter than 6 femtosecond have been created directly from a Ti:sapphire laser oscillator with the help of special dispersion-compensating mirrors.

To understand the mode structure of a femtosecond frequency comb and the techniques applied for its stabilization one can look at the idealized case of a pulse circulating in a laser cavity with length L as a carrier wave at f_c that is

subject to strong amplitude modulation described by envelope function $A(t)$. The function defines the pulse repetition time $T=fr^{-1}$ by demanding $A(t)=A(t-T)$ where T is calculated from the cavity mean group velocity: $T=2L/v_g$. The pulses however are not necessarily identical. This is because the pulse envelope $A(t)$ propagates with v_g while the carrier wave travels with its phase velocity. As a result the carrier shifts with respect to the pulse envelope after each round trip by a phase angle as shown in Figure.4. Unlike the envelope function, which provides us with a more rigorous definition of the pulse repetition time $T=fr^{-1}$, the electric field is, in general, not expected to be periodic in time. Because of the periodicity of the envelope function the electric field at a given place can be written as

$$E(t) = A(t)e^{2\pi f_c t} + c.c = \sum_q A_q e^{-2\pi i(f_c + qf_r)t} + c.c. \tag{9}$$

As the envelope function $A(t)$ is strictly periodic it has been written as a Fourier series

$$A(t) = \sum_q A_q e^{-2\pi i q f_r t} \tag{10}$$

Where A_q are Fourier components of $A(t)$. Equation 9 shows that, under the assumption of a periodic pulse envelope, the resulting spectrum represents a comb of laser frequencies separated by the pulse repetition frequency f_r . Since f_c is not necessarily an integer multiple of f_r the modes are shifted from being exact harmonics of the pulse repetition frequency by an offset $f_o < f_r$:

$$f_n = n f_r + f_o \tag{11}$$

with a large (10^6) integer n . This equation maps two radio frequencies f_r and f_o onto the optical frequencies f_n . While f_r is readily measurable and usually lies between a few 10 MHz and a few GHz depending on the length of the laser resonator, f_o is not easy to access unless the frequency comb contains more than an optical octave. The intuitive picture given here can even cope with a frequency chirp, i.e. a carrier frequency that varies across the pulse. In this case the envelope function becomes complex in value and the comb structure derived above stays valid provided the chirp is the same for all the pulses. Under this assumption, which is reasonable for a stationary pulse train, $A(t)$ remains a periodic function. In the time domain the frequency offset is obvious because the group velocity differs from the phase velocity inside the cavity and therefore the carrier wave does not repeat itself after one round trip but appears phase shifted by $\Delta\phi$ as shown in Figure.4. The offset frequency is then calculated from $f_o = \Delta\phi / (2\pi T)$.

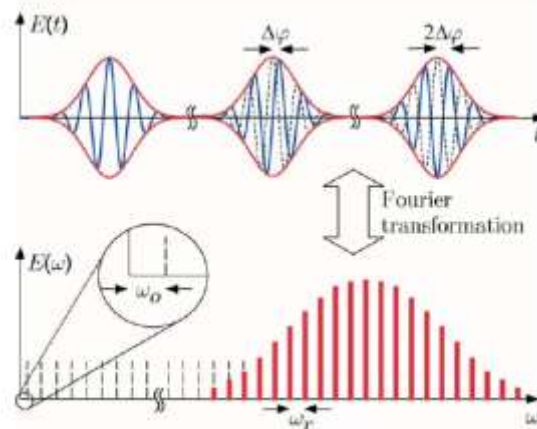


Figure4: Pulse train emitted by a mode locked laser and the corresponding spectrum.

One might argue that no laser has line width zero and that one should treat the carrier not as a ideal single frequency wave f_c but as a source with general line width function $C(t)$. Even if no technical noise would be present, there would still be some sort of fundamental Schawlow-Townes limit connected with the line width of each mode. As long as we still have the periodicity of $A(t)$ Equation. 3.2.1 reads then as

$$E(t) = A(t)C(t) + c.c. \tag{12}$$

Fourier transforming $E(t)$ brings us into the frequency domain and back ($\omega=2\pi f$):

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} E(t)e^{j\omega t} dt, E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} E(\omega)e^{-j\omega t} d\omega \tag{13}$$

With the help of the convolution theorem

$$\sqrt{2\pi}A(t)C(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} (A(t) \otimes C(t))e^{-j\omega t} d(\omega) \quad (14)$$

$$E(\omega) = \frac{1}{\sqrt{2\pi}} (A(\omega) \otimes C(\omega)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(\omega')C(\omega - \omega')d\omega' + c.c. \quad (15)$$

We get

$$A(\omega) = \sqrt{2\pi} \sum_{n=-\infty}^{+\infty} A_n \delta(\omega - n\omega_r) \quad (16)$$

The Fourier transforms of A(t) and C(t) are given by

$$C(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} C(t)e^{j\omega t} dt \quad (17)$$

So therefore

$$E(\omega) = \sum_{n=-\infty}^{+\infty} A_n C(\omega - n\omega_r) + c.c. \quad (18)$$

This sum represents a periodic spectrum in frequency space with periodicity $f_r = 1/T$. The mode shape function is duplicated by the strong amplitude modulation induced by Kerr lens mode locking. Assuming the simplified case of a carrier wave $C(t) = e^{-2\pi j f_c t}$ brings us back to equation 9. A chirp of the pulse may be hidden in the complex Fourier components A_n . Note that the only assumption necessary to create a precisely equidistant comb is the periodicity of the envelope function. In the time domain, the output of a mode-locked femtosecond laser may be considered as a continuous carrier wave that is strongly amplitude modulated by a periodic pulse envelope function. The beat note between the carrier wave and the CW oscillator is observed in a stroboscopic sampling scheme, if such a pulse train and the light from a CW laser are combined on a photo detector. The detector signal will therefore reveal a slow modulation at the beat frequency modulo the sampling rate or pulse repetition frequency.

The important fact to learn from this section is that such a femtosecond frequency comb has two degrees of freedom which are the repetition frequency f_r and the offset frequency $f_o < f_r$. Depending on the application one or both degrees of freedom have to be stabilized. Furthermore the fast amplitude modulation of the Kerr lens keeps the inter-mode spacing constant even across a vast spectrum of modes. The mode-locked laser based OFCG fixed comb spacing due to its cavity length with stability depending upon mode locking condition. As the spectral width of these pulsed lasers scales inversely with the pulse duration the advent of femtosecond lasers has opened the possibility to directly access THz frequency gaps.

Fiber Ring Based OFCG Configuration:

The fiber ring based OFCG was first proposed by Ho and Kahn. Pioneering work was carried out by Seeds' group at UCL, who successfully demonstrated a THz comb span from a fiber ring based OFCG [5].

The operation of the fiber ring based OFCG is based on using a resonant cavity to enhance the phase modulation of the light to help spread the sideband spectrum widely. Due to the multiple passes through the electro optical modulator (EOM), pulse trains are formed within the cavity and an optical comb is produced as a result. Due to its exceptionally long cavity compared to other types of OFCG, matching the cavity length such that its resonances coincide with both the reference light wave frequency and the RF reference presents a major challenge.

Without this matching, the comb generation will be neither efficient nor stable. The UCL group has adopted a wide line width laser source, operating the OFCG in multiple cavity modes to help stabilize the comb; thus, each of the comb lines is composed of several cavity modes. The alternative approach is actively tracking the reference laser frequency drift by adjusting the cavity length [6], keeping the comb in stable single mode operation.

Fiber ring based OFCG that consists of an optical amplifier, an optical phase modulator, fiber delay lines and optical couplers, as shown in Figure 5. Isolators are used to ensure that the light propagates only in one direction, and to avoid unwanted reflections. The phase modulator is polarization dependent and packaged with PM fiber, so a polarization controller is required before it. An additional polarizer can be used to suppress the unwanted polarization and to ensure that the output comb has a stable polarization. With the optical delay line in the cavity, the cavity length of the OFCG can be adjusted and controlled so that the free spectral range of the cavity is almost equal to 5 MHz. All these optical components are placed within an enclosed box to isolate them from ambient acoustic noise and thermal drift.

Table 1. Comparison of different parameters for generation of Optical Frequency Comb Generator

Parameters	EO Phase Modulation	Fiber Ring Based OFCG	Femtosecond
Spacing Between the comb lines	Directly proportion to the RF frequency.	Path lengths, the phase relationship between the beat signal and the LO signal.	Constant due to fast amplitude modulation of the Kerr lens.
Cavity Length	Not dependent on cavity length.	Optical delay line in the cavity it is kept to 5 MHz(Tunable Length)	Kerr lens Mode locking condition(Fixed Length)
No. of comb lines dependent on	Linearly related with RF voltage	Not Applicable	Repetition rate inside the cavity

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